Homogenization process for a non linear monotone problem with L^1 data

Silvio Bove

Abstract

Many physical and engineering systems involve materials or domains with extremely fine microstructures, such as porous media, composite materials, or surfaces with rapidly oscillating boundaries. Solving a partial differential equation in such a heterogeneous domain is often analytically and numerically intractable. Homogenization provides a way out: it is a mathematical technique that replaces the complex microscopic geometry with an effective macroscopic model, capturing the averaged influence of the small-scale structures.

In this talk, we consider a nonlinear monotone elliptic equation posed in a highly oscillating domain of "brush type", a three dimensional geometry made of a fixed base from which numerous thin vertical teeth emerge. The diameter of each tooth is of order ε, their arrangement is not assumed to be periodic, and their cross-sections may vary and need not be smooth. The model we study follows the nonlinear monotone setting introduced in [BCG99], while the geometric configuration corresponds to the framework of [GGM17].

We study the asymptotic behaviour, as $\varepsilon \to 0$, of our nonlinear monotone elliptic problem when the source term belongs to $L^1(\Omega)$. Since the data is too weak to apply the usual variational techniques, we work within the framework of renormalized solutions, as developed in [GGM17], defined for a degenerate elliptic equation (only the vertical derivative is involved in the upper part of the domain).

We will describe the homogenization process for this class of equations and show that the solution u_ε of the ε–dependent problem converges, as $\varepsilon \to 0$, to the solution of a suitable limit problem.

This presentation is based on joint work with Olivier Guibé.

References

[BCG99] D. Blanchard, L. Carbone, and A. Gaudiello. Homogenization of a monotone problem in a domain with oscillating boundary. *M2AN, Math. Model. Numer. Anal.*, 33(5):1057–1070, 1999.

[GGM17] A. Gaudiello, O. Guibé, and F. Murat. Homogenization of the brush problem with a source term in L^1. *Arch. Ration. Mech. Anal.*, 225(1):1–64, 2017.