OPTIMALITY OF THE HEXAGONAL TILING FOR THE TORSIONNAL RIGIDITY AND THE FIRST DIRICHLET EIGENVALUE WITH FIXED INRADIUS IN 2D

ALEXIS DE VILLEROCHÉ

In this talk, I will present ongoing work with Dorin Bucur and Giuseppe Buttazzo about optimality of the hexagonal tiling for some problems under inradius constraint in two dimensions. We are interested in showing that the set $\Omega_{hex} = \mathbb{R}^2 \setminus Z_{hex}$, where Z_{hex} is the set of the centers of the hexagonal tiling, is a solution to the problems,

$$\begin{aligned} &\min \big\{ \lambda_p(\Omega) \;\big|\; \Omega \subset \mathbb{R}^2,\ \rho(\Omega) = 1 \big\}, \\ &\sup \left\{ \frac{T_p(\Omega)}{|\Omega|} \;\middle|\; \Omega \subset \mathbb{R}^2,\ \rho(\Omega) = 1 \right\}, \end{aligned} \tag{1}$$

with $2 < p < +\infty$, where $\lambda_p(\Omega)$ and $T_p(\Omega)$ represent respectively the principal eigenvalue of the p-Laplacian with Dirichlet boundary conditions and the p-torsionnal rigidity of Ω. In order to gain information about (1), we actually focus on simpler problems defined in the class

$$\mathcal{A}_\varepsilon = \big\{ \Omega \subset \mathbb{R}^2,\ \Omega = \mathbb{R}^2 \setminus \bigcup B(x_i, \varepsilon),\ \text{where } (x_i) \text{ has no accumulation points} \big\},$$

and we partially solve the following problems

$$\begin{aligned} &\min \big\{ h(\Omega) \;\big|\; \Omega \in \mathcal{A}_\varepsilon,\ \rho(\Omega) = 1 \big\}, \\ &\min \big\{ \lambda_2(\Omega) \;\big|\; \Omega \in \mathcal{A}_\varepsilon,\ \rho(\Omega) = 1 \big\}, \\ &\sup \left\{ \frac{T_2(\Omega)}{|\Omega|} \;\middle|\; \Omega \in \mathcal{A}_\varepsilon,\ \rho(\Omega) = 1 \right\}, \end{aligned}$$

where $h(\Omega)$ is the Cheeger constant of Ω.