Discrete surfaces: a geometric measure theory perspective

Continuous definitions (such as those of surface, regularity, dimension, curvatures ...) generally cannot be readily given a discrete counterpart. Moreover, this discrete counterpart is generally not unique and highly scale-dependent. There are multiple ways of developing a theory for discrete surfaces and the choice of an appropriate framework is directly related to the kind of discrete data we aim to process, and for which purpose i.e. the kind of surfaces we try to model.

We propose to focus on unstructured data in the sense that we do not have any underlying parametrization or topological information associated with our data, a typical example being point cloud data (e.g. obtained from scan acquisition) or different kinds of diffuse approximations (as in MRI for instance). Our motivation is twofold: first, a large range of data-types initially come without any parametrization information. Moreover, let us point out that the construction of such a parametrization (for instance the definition of a triangulation starting from a point cloud) is a challenging active topic in itself, and a better understanding of unstructured data is an essential pre-processing step.

Geometric measure theory offers a particularly well-suited framework for the study of such unstructured discrete surfaces. The long-standing Plateau problem has given birth to several different weakenings of the notion of surface. While their common purpose was to gain compactness while preserving mass/area continuity (or lower semi-continuity at least), they actually provide consistent settings for developing a theory of discrete surfaces, as we intend to explain in this talk, focusing on so-called "varifolds".