

Sharp Makai-type inequalities for principal frequencies

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In this talk we will present sharp lower bounds for the Poincaré–Sobolev constants $\lambda_{p,q}(\Omega)$ over bounded convex open sets $\Omega \subseteq \mathbb{R}^N$. More precisely, we will show that $\lambda_{p,q}(\Omega)$ can be bounded from below by the p -th power of the ratio between the perimeter of Ω and a suitable power of its volume, with an explicit optimal constant. This generalises a classical result for torsional rigidity due to Makai in the planar case. A key ingredient of the proof consists of new geometric optimal bounds for the Lebesgue norms of the distance function from the boundary, which are of independent interest. As an application, we will give a complete picture of the sharp inequalities for $\lambda_{p,q}(\Omega)$ in terms of suitable powers of perimeter, inradius and volume of Ω , and in particular we will confirm an open conjecture by Briani, Buttazzo and Prinari on the p -torsional rigidity in arbitrary dimension. The talk is based on a joint work with Francesca Prinari (Università di Pisa).