Title: Generalized Wasserstein Barycenters

Abstract: Optimal transport provides a natural way to compare probability distributions. Agueh and Carlier in particular introduced a model for computing barycenters with respect to the Wasserstein distance that found numerous applications in data science and image processing. This is conceived as a Fréchet mean, namely a variational problem where one minimizes the weighted sum of the distances squared from N given measures. I will discuss a generalization of this model to the case where one allows negative weights, so as to extrapolate rather than interpolate. In the two-measures case this provides a way to extend geodesics. The difficulty comes from the lack of standard convexity, contrary to the barycenter model of Agueh and Carlier. Surprisingly, in the case with only one positive weight the problem enjoys some hidden convexity properties which allow to rewrite it in equivalent convex formulations and to fully characterize (unique) minimizers. In particular, the problem can be recast as an instance of weak (multimarginal) optimal transport. One can take advantage of this formulation to devise an efficient approach to approximate the solution on point clouds, via entropic regularization and a variant of the Sinkhorn algorithm. This talk is based on joint works with Thomas Gallouët and Andrea Natale.